Invoking the Lamé constants for an isotropic, linearly elastic solid, one finds

$$dW_{PD} = \frac{4}{3} V\tau(d\epsilon_x - \frac{d\tau}{\mu}) \qquad (6)$$

(Duvall, 1972). Here μ is a Lamé constant or the shear modulus and $\tau \equiv (\sigma_x - \sigma_y)/2$, the maximum resolved shear stress.

No measurements of the Hugoniot elastic limit (HEL) for silver have been published. For a solid whose yielding response under shock shows no relaxation phenomena (i.e., no strain rate effects) one can estimate the HEL from the static yield stress in tension Y. The von Mises or Tresca yield criterion in uniaxial strain is $-\tau = Y/2$. For the linearly elastic solid the HEL is given by

$$-\sigma_x = Y\frac{(1-\nu)}{(1-2\nu)} = P_x^{HEL}$$

where ν is Poisson's ratio and $P_x \equiv -\sigma_x$ (Duvall, 1972).

For commercially pure silver the Metals Handbook lists a yield stress in tension of 0.54 kbar. For Poisson's ratio it lists 0.37 for annealed silver and 0.39 for hard drawn. (Wise and Cox, 1961). Dawson's work (1965b) on 99.999% pure, annealed silver gives a value of yield stress of 0.42 kbar. Values of 0.5 kbar for Y and 0.37 for ν were used in computations.

We than find $P_x^{HEL} = 1.2$ kbar. Provided that no volume changes result from plastic strain, one can derive $-\overline{P} = \sigma_x + \frac{2}{3}Y$ (Fowles, 1961). For the equation of state calculations in this work, this correction to the pressure (0.3 kbar for silver) was neglected.

The agreement of HEL's calculated as above with experimental values is not very good. A comparison done by Duvall

(1961) for iron and steels show experimental values to be about twice as large as calculated ones. Work in annealed mild steel showed observed HEL's to be two to three times calculated values (Jones, Nielson, and Benedick, 1962; Taylor and Rice, 1963). Fowles' work (1961) on 2024 aluminum, both hardened and annealed, found agreement to within the precision of the experiment. Other work on Dural aluminum found experimental HEL's 50% higher in hardened material and 150% higher in the annealed case (Taylor and Whiffen, 1948). It should be noted that these HEL values correspond to values after several millimeters of shock propagation.

D. Work of Plastic Deformation

Assuming $-\tau = Y/2$ we may write Eq. (6) as

$$dW_{PD} = -\frac{2}{3} Y\, V(d\epsilon_x + dY/2\mu) \qquad (7)$$

If there is no work hardening so that yield strength in tension is constant the plastic work is

$$W_{PD} = -\frac{2}{3} Y_o V_1 \left(\frac{V}{V_1} - 1\right)$$

where V_1 is the volume at the Hugoniot elastic limit. Work hardening may be included by allowing yield strength in tension to be a function of plastic work. Yield strength in tension (condition of uniaxial stress) as a function of strain has been published for silver by Dawson (1965). One can find the uniaxial strain for equivalent plastic work by equating plastic work as a function of strain in the two cases. The approximate result is

$$\epsilon_x \approx -\frac{3}{2} \epsilon_s \qquad \text{(Fowles, 1961).}$$